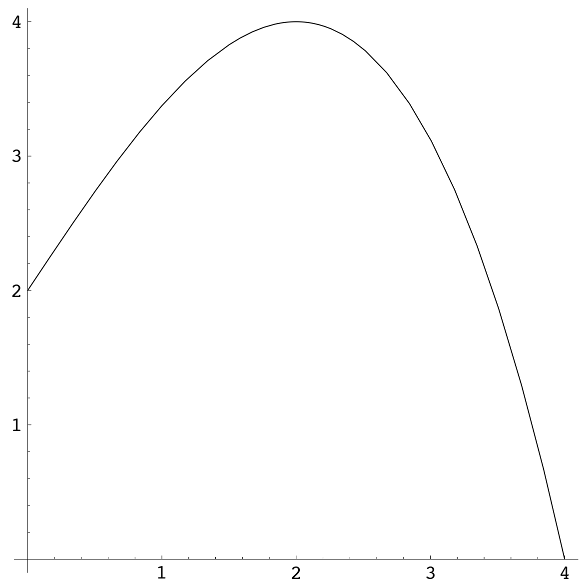
Worksheet 14, Math 10560

1 Use the trapezoidal rule with step size $\Delta x = 2$ to approximate the integral $\int_0^4 f(x)dx$ where the graph of the function $f(x)$ is given below.



2 Use Simpson's rule with step size $\Delta x = 1$ to appoximate the integral $\int_0^4 f(x)dx$ where a table of values for the function $f(x)$ is given below.

x	0	1	2	3	4
$f(x)$	2	1	2	3	5

3 (a) Circle the letter below alongside the trapezoidal approximation to

$$\ln 3 = \int_1^3 \frac{1}{x} dx \quad \text{using} \quad n = 8$$

A $\displaystyle \int_1^3 \frac{1}{x} dx \approx \frac{1}{8}\Big[1 + 2\Big(\frac{4}{5}\Big) + 2\Big(\frac{2}{3}\Big) + 2\Big(\frac{4}{7}\Big) + 2\Big(\frac{1}{2}\Big) + 2\Big(\frac{4}{9}\Big) + 2\Big(\frac{2}{5}\Big) + 2\Big(\frac{4}{11}\Big) + \Big(\frac{1}{3}\Big)\Big]$

B $\displaystyle \int_1^3 \frac{1}{x} dx \approx \frac{1}{12}\Big[1 + 4\Big(\frac{4}{5}\Big) + 2\Big(\frac{2}{3}\Big) + 4\Big(\frac{4}{7}\Big) + 2\Big(\frac{1}{2}\Big) + 4\Big(\frac{4}{9}\Big) + 2\Big(\frac{2}{5}\Big) + 4\Big(\frac{4}{11}\Big) + \Big(\frac{1}{3}\Big)\Big]$

C $\displaystyle \int_1^3 \frac{1}{x} dx \approx \frac{1}{8}\Big[1 + \Big(\frac{4}{5}\Big) + \Big(\frac{2}{3}\Big) + \Big(\frac{4}{7}\Big) + \Big(\frac{1}{2}\Big) + \Big(\frac{4}{9}\Big) + \Big(\frac{2}{5}\Big) + \Big(\frac{4}{11}\Big) + \Big(\frac{1}{3}\Big)\Big]$

(b) Recall that the error E_T in the trapezoidal rule for approximating $\int_a^b f(x)dx$ satisfies

$$\Big| \int_a^b f(x)dx - T_n \Big| = |E_T| \leq \frac{K(b-a)^3}{12n^2}$$

whenever $|f''(x)| \leq K$ for all $a \leq x \leq b$.

Use the above error bound to determine a value of n for which the trapezoidal approximation to $\ln 3 = \int_1^3 \frac{1}{x} dx$ has an error

$$|E_T| \leq \frac{1}{3} 10^{-4}.$$

4 Suppose the Midpoint rule is to be used to approximate the integral

$$\int_0^{10} \sin(\sqrt{6}\, x)\, dx\ .$$

What is the minimum number of points required to guarantee an accuracy of $1/1000$?

500

550

600

650

450

5 Use the Trapezoidal rule with step size $\Delta x = 1$ to appoximate the integral $\int_0^4 f(x)dx$ where a table of values for the function $f(x)$ is given below.

x	0	1	2	3	4
$f(x)$	2	1	2	3	5

6 Consider the integral

$$\int_0^2 (2x+3)\ dx.$$

(a) (5 pts.) Evaluate this integral exactly.
(b) (8 pts.) Using the Trapezoidal Rule with $n = 4$ find an approximation to the integral.
(c) (2 pts.) Explain your answer in part (b).**Hint:**Consider the error.

7 Suppose that $\left|f''(x)\right| \leq 1$ for $0 \leq x \leq 2$. If E_M is the error in the Midpoint Rule using n subintervals, then $|E_M|$ is less than

$\frac{1}{3n^2}$

0

$\frac{1}{12n^2}$

$\frac{2}{3n^2}$

$\frac{1}{24n^2}$

8 The integral $\displaystyle\int_1^3 \frac{dx}{x}$ is estimated using the Trapezoidal Rule, using subintervals of size $\Delta x = 1$. The approximation to $\ln 3$ obtained is